39. The angular positions of the first-order diffraction lines are given by $d\sin\theta = \lambda$. Let λ_1 be the shorter wavelength (430 nm) and θ be the angular position of the line associated with it. Let λ_2 be the longer wavelength (680 nm), and let $\theta+\Delta\theta$ be the angular position of the line associated with it. Here $\Delta\theta = 20°$. Then, $d\sin\theta = \lambda_1$ and $d\sin(\theta+\Delta\theta) = \lambda_2$. We write $\sin(\theta+\Delta\theta)$ as $\sin\theta\cos\Delta\theta + \cos\theta\sin\Delta\theta$, then use the equation for the first line to replace $\sin\theta$ with λ_1/d, and $\cos\theta$ with $\sqrt{1-\lambda_1^2/d^2}$. After multiplying by d, we obtain

$$\lambda_1\cos\Delta\theta + \sqrt{d^2-\lambda_1^2}\,\sin\Delta\theta = \lambda_2\ .$$

Solving for d, we find

$$\begin{aligned} d &= \sqrt{\frac{(\lambda_2-\lambda_1\cos\Delta\theta)^2+(\lambda_1\sin\Delta\theta)^2}{\sin^2\Delta\theta}} \\ &= \sqrt{\frac{[(680\,\text{nm})-(430\,\text{nm})\cos 20°]^2+[(430\,\text{nm})\sin 20°]^2}{\sin^2 20°}} \\ &= 914\,\text{nm} = 9.14\times10^{-4}\ \text{mm}\ . \end{aligned}$$

There are $1/d = 1/(9.14\times10^{-4}\,\text{mm}) = 1090$ rulings per mm.