64. Consider two light rays crossing each other at the middle of the lens (see Fig. 37-42(c)). The rays come from opposite sides of the circular dot of diameter D, a distance L from the eyes, so we are using the same notation found in Sample Problem 37-6 (which is in the textbook supplement). Those two rays reach the retina a distance L' behind the lens, striking two points there which are a distance D' apart. Therefore,

$$\frac{D}{L} = \frac{D'}{L'}$$

where $D = 2\,\text{mm}$ and $L' = 20\,\text{mm}$. If we estimate $L \approx 450\,\text{mm}$, we find $D' \approx 0.09\,\text{mm}$. Turning our attention to Fig. 37-42(d), we see

$$\theta = \tan^{-1}\left(\frac{\frac{1}{2}D'}{x}\right)$$

which we wish to set equal to the angle in Eq. 37-12. We could use the small angle approximation $\sin\theta \approx \tan\theta$ to relate these directly, or we could be "exact" – as we show below:

$$\text{If} \quad \tan\phi = \frac{b}{a} \ , \quad \text{then} \quad \sin\phi = \frac{b}{\sqrt{a^2+b^2}} \quad .$$

Therefore, this "exact" use of Eq. 37-12 leads to

$$1.22\frac{\lambda}{d} = \sin\theta = \frac{\frac{1}{2}D'}{\sqrt{x^2 + (D'/2)^2}}$$

where $\lambda = 550 \times 10^{-6}\,\text{mm}$ and $1\,\text{mm} \le x \le 15\,\text{mm}$. Using the value of D' found above, this leads to a range of d values: $0.015\,\text{mm} \le d \le 0.23\,\text{mm}$.